Achievements, Fields of Interest

My fields of scientific interests mostly contain theory of semigroups of operators, cosine operator-function theory, differential equations in Banach spaces, numerical analysis in linear and semilinear differential equations in abstract spaces, approximation of the spectrum of operators. Let me describe the most interesting to my mind points of my research.

* The stable and regular consistency of closed linear operators on general discretisation scheme for approximation of spectrum of operators are investigated. It is shown that for unbounded linear operators regular consistent implies coincidence of dimensions of generalized eigenvector spaces of corresponding eigenvalues. So the eigenvalue is stable in Kato's sense iff in the neighborhood of this eigenvalue one has regular consistency of operators.

* It is established that compact convergence of resolvents implies regular consistent of closed operators on the whole complex plane.

* It is developed approach of checking regular convergence of operators by using the Ahues's type conditions, which are easily applied in practice. For instance, it is shown in principle that convergence $\|(B_n - B)B_n^q\| \to 0$ as $n \to \infty$ for some q implies that $\lambda - B_n \to \lambda - B$ regularly.

* Investigated the simplest difference schemes for solving Cauchy problems for the first and the second order equations in a Banach space E

$$u'(t) = Au(t), u(0) = u^0, \tag{1}$$

$$u''(t) = Au(t), u(0) = u^0, u'(0) = u^1, \tag{2}$$

For semidiscrete case, i.e. for approximation in space variable

$$u_n'(t) = A_n u_n(t), u_n(0) = u^0,$$

$$u_n''(t) = A_n u_n(t), u_n(0) = u_n^0, u_n'(0) = u_n^1,$$

it is established Trotter-Kato's version theorem in the general discretisation scheme for C_0-semigroups and for analytic semigroups. For full discretisation scheme it is well known that in general for the C_0-semigroups and the forward scheme we have the following stability condition

$$\tau\|A_n^2\| = O(1).$$

In the case of analytic semigroups for the forward scheme obtained the following stability condition

$$\tau\|A_n\| < 1/(M+2),$$

which can't be improved even in Hilbert spaces for self-adjoint operators. In the case of almost periodic semigroups and the forward scheme for differential equation of the first order in time (1) obtained the stability condition

$$\tau\|A_n\| < 1.$$

To consider second order case (2) we introduce the notion of discrete cosine operator-functions and develop the theory of convergence of discrete cosine operator functions. It is proved that condition $\tau\|A_n\| < 4$ is necessary and sufficient for the stability of the forward difference scheme for solving problem (2) with almost periodic solutions.

* Discovered stability condition of the forward scheme for the positive C_0 -semigroups (first order equation in time) in the form $\tau\|A_n\| < 1$. In formulation of the Trotter-Kato's version theorem for positive semigroups instead of stability estimate on semigroups we use the condition of positive off diagonal property for infinitesimal generators of semigroups.

* For the Cauchy problem in the space $C^\theta(\Omega)$ with the parabolic equation in the form

$$u_t'(x,t) = \Delta_x u(x,t) + f(x,t),$$

full discretisation schemes are considered. It is established that in general Crank-Nicolson scheme is not maximal regularly stable for the functions from the set of functions

$\Xi = B_\tau(0,T;C_h^\theta(\Omega_h)) \cap C_\tau(0,T;C_h(\Omega_h))$. For such functions Crank-Nicolson scheme is only unconditionally stable. The maximal regularity stability for the functions from Ξ is proved for the implicit schemes and also for the case of Crank-Nicolson scheme, but under assumption on relation of steps discretisation in time and space.

* Investigated the property of almost periodicity of cosine (COF) and sine operator-functions. It is proved that COF $C(t,A)$ is almost periodic iff $C(t,A)$ is uniformly bounded, $\sigma(A) \subseteq R_-$ and the set of all linear combinations of eigenvectors of operator A is dense in E. Each generalized solution of Cauchy problem (2) is almost periodic iff $C(t,A)$ is almost periodic and $0 \in \rho(A)$.

* Investigated the property of compactness of cosine and sine operator functions. In particular, it is shown that $C(t,A) - I$ is compact for every $t > 0$ iff the generator A is compact.

* The asymptotic behavior of COF $C(t,A)$ investigated in terms of the behavior of resolvent and also behavior of semigroup $exp(tA)$ near imaginary axis. In particular, we have proved that if $\|exp(zA)\| \le M_1(\frac{|z|}{Rez})^{1/2}$ for any $Rez > 0$, then $\|C(t,A)x\| \le M_2(1+t^2)\|Ax\|$ for any $x \in D(A)$.

* Investigated the local C-cosine families. the generation theorem for the local C-cosine families is obtained. Connections with the local n-times integrated semigroups has been studied.

* It is shown that the spectral condition $0 \in \rho(A) \cup C\sigma(A)$ is necessary and sufficient for β-Cesaro stability of bounded C_0-semigroup on a Banach space E. Analogous results are valid for other averages of semigroups and cosine operator functions.

* Asymptotical behaviour of semigroups (and others resolution families) are considered. Let A be the generator of a tempered C_0-semigroup acting on a Banach space E. Let $\alpha > 0$ and suppose that there exists an operator $B \in B(E)$ such that

(i) $\lambda^{\alpha+1}(\lambda - A)^{-1} \to B$ in the strong operator topology as $\lambda \to 0^+$ in R;

(ii) there exists $C > 0, N \ge 0$ and $\rho_0 > 0$ such that

$$\|\rho^{1+\alpha}(\rho e^{i\phi} - A)^{-1}\| \le \frac{C}{\cos^N \phi}, \quad 0 < \rho \le \rho_0,\ \phi \in (-\pi/2, \pi/2).$$

Conditions (i) and (ii) are necessary and sufficient for existence of positive integer m such that

$$\lim_{t\to\infty} \frac{1}{t^{m+\alpha}} \int_0^t (t-s)^{m-1} e^{As}\, ds = B$$

in the strong operator topology.

The same is proved for cosine operator-functions.

* Investigated approximations of the problems (1) and (2) by rational approximation (they have a more complicated difference schemes, for example, Pade fractions) and the optimal order of convergence and stability conditions are obtained. In particular, it is shown that Crank-Nicolson scheme for analytic semigroups is stable in a general Banach space E.

* The Richardson extrapolation methods are investigated for increasing the accuracy of approximation of the solution of the Cauchy problems (1) and (2) in a Banach space using diagonal Pade fractions.

* Discretisation of an evolution equation with analytic semigroup and backwards time (ill-posed problem for (1)) are investigated. We consider regularization methods and obtain the error estimates of the methods which are can't be improved even in self-adjoint case.

* The approximation of ill-posed Cauchy problem by using C-semigroups is established. More exactly, if in (1) the operator $-A$ generates analytic semigroups, then the solution is given by some C-semigroup $u(t) = C^{-1}S(t)u^0$. We proved Trotter-Kato Theorem for the local C-semigroups, and some discrete semigroups approximation theorems are established. It is also shown that stochastic differential equation

$$du_\alpha(t) = -Au_\alpha(t)dt - \alpha Au_\alpha(t)dw(t),\ u(0) = u^0,$$

where $w(\cdot)$ is a standart Wiener process, gives us stochastic regularisation of (1) with $-A$ which generates analytic semigroup. Discretisation in time, i.e. simplest time discretisation scheme, is also considered. More precisely, we established that the scheme

$$U_{n,\alpha}(t+\tau_n) - U_{n,\alpha}(t) = -\tau_n A_n U_{n,\alpha}(t) - \alpha\,\Delta w(t) A_n U_{n,\alpha}(t), U_{n,\alpha}(0) = I_n,$$

is stable and $U_{n,\alpha_n}(t)u_n^0 \to u(t)$ as $n \to \infty$, i.e.

$$\sup_{\|u_n^0 - p_n u^0\| \leq \delta} \|U_{n,\alpha_n}(t)u_n^0 - p_n u(t)\| \to 0 \text{ as } \delta_n \to 0.$$

Stability of the scheme is meant in the sense:

$$\eta_n := \sup\left\{E\left[\|U_{n,\alpha_n}(t)u_n^0 - u_{n,\alpha_n}(t)\|\right] : \|u_n^0\| \leq 1\right\} \to 0,$$

and convergence of method is meant in the following sense: $E\left[\|U_{n,\alpha_n}(t)u_n^0 - p_n u(t)\|\right] \leq C\sqrt{\tau_n}\|A\exp(-TA)u^0\| + \|u_{n,\alpha_n}(t) - p_n u_{\alpha_n}\| + C\eta_n\|u_n^0\|, \quad 0 < t \leq T$. Here we suppose that $\limsup_{n\to\infty} k_n\alpha_n^2 < C_1 t$, and there are constant C such that $\tau_n\|A_n^2\|e^{c\|A_n^2\|} \leq C$.

* Investigated the Cauchy problem

$$u'(t) = Au(t) + f(t, u(t)), \quad u(0) = u^0,$$

with analytic semigroups $exp(tA)$ and function f which is smooth enough in its arguments. Using theory of rotation of the vector field under condition of compactness of resolvent we establish the convergence of the difference schemes and obtain order of convergence. The same is done for the second order equation in time. Under additional assumption of positiveness in a Banach lattice E or of Ljapunov stability condition one also obtains the approximations of the **stable** periodic solutions of the semilinear equation $u'(t) = Au(t) + f(t, u(t))$.

* Investigated perturbation of C_0-semigroups by multiplicative case. It is shown, in particular, that the semigroups satisfy $\|exp(tA) - exp(tC)\| = O(t)$ as $t \to 0$ iff $C = A(1 - \lambda\hat{U}(\lambda)) + \lambda^2\hat{U}(\lambda)$ for some Lipschitz continuous step responses function $U(t)$, i.e. the function which satisfy the equation

$$U(t+s) - U(t) = exp(tA)U(s) \ for \ t, s \geq 0, U(0) = 0.$$

The same is done for cosine operator functions. We also proved that perturbations with cumulative outputs give us all known additive perturbations of generator of COF. Perturbations with the step response for COF have not bee considered before at all.

* We investigate also the continuity, measurability, compactness, positivity and almost periodicity properties of step responses functions and of cumulative outputs functions for semigroups and COF and also its asymptotic behavior.

* We characterize in terms of semivariation such cosine operator functions $C(\cdot)$ that for each continuous function f the function $\int_0^t \int_0^{t-s} C(\tau)f(s)d\tau\, ds$, $t > 0$, is twice continuously differentiable (MR-property). We also give a new characterization of those $C(\cdot)$ which have bounded generators. It is also proved that if A generates a cosine operator function of bounded semivariation, then operator A is bounded.

* In Hilbert space H it was established that perturbations in the case of C_0-groups and cosine operator functions are described by step response functions, which have semivariations $CV(U(\cdot), t), CV(F(\cdot), t)$ of order not less than $O(\sqrt{t})$ as $t \to 0$.

* It is proved that Cauchy problem for the second order equations in Banach spaces is coersive in $C([0, T]; E)$ or in $L^p([0, T]; E)$ spaces iff operator A is bounded.

* Discrete maximal regularity estimates for parabolic equations with elliptic operator of second order in $C^\alpha(\Omega)$ spaces established. Let us note that in $C^\alpha(\Omega)$ space elliptic operator does not generate C_0-semigroup. Consider the following discrete scheme

$$\frac{U^n - U^{n-1}}{\tau} = A(\frac{U^n + U^{n-1}}{2}) + F^n, \ n \in \{1, ..., K\}, U^0 = u_0,$$

where operator A generates analytic semigroup, but not C_0-semigroup in $C^\alpha(\Omega)$ space. Let $G \in B(\{0, 1, ..., N_1\}; \tilde{E})$ be such that $G^0 = 0$ and consider the problem with restriction $F^n = RG^n$ for $n = 1, ..., N_1$ and $u_0 = 0$. Then there exists a unique solution $U \in B\Big(\{0, 1, ..., N_1\}; E\Big)$ and for any $n = 0, 1, ..., N_1$

$$\|U^n\|_{2+2\theta} \leq c\Big(\max_{0 \leq n \leq N_1} \|G^n\|_{2\theta} + \max_{0 \leq n_1 < n_2 \leq N_1} ((n_2 - n_1)\tau)^{-\theta} p(G^{n_2} - G^{n_1})\Big),$$

where p is some seminorm and the norms are discrete analogy of $C^\alpha(\Omega)$ space norms.

* Discrete maximal regularity inequality for Cauchy problem in $L^p_{\tau_n}(R; E)$ space is considered. For the general Banach space E the inequality with logarithm is obtained. We established discrete maximal regularity inequality for implicite scheme and also for Crank-Nicolson scheme in case of E to be UMD space under condition of R-boundedness of resolvent.

* Developed approach to approximate the perturbed C_0-semigroups and cosine operator functions (analogy of Trotter-Kato theorem) using infinitesimal properties of step responses and cumulative outputs functions.

* Developed approach to approximate the perturbed C_0-semigroups and cosine operator functions by discretisation in time using information on step responses and cumulative outputs functions near the zero or using the information on Laplace transform of them.

* Investigated the method of conjugate gradients for adaptive spatial filtering. First we find the proper preprocessor, which reduces the dimensions of the problem in two times without increasing the conditionally number. The second, a comparison of the proposed adaptive processing technique with the existing gradient methods has shown that the rate of convergence exceeds the rate of convergence of ordinary gradient algorithms by a factor of more than 2.